

# *Algebra II Standards*

## **Algebra II**

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Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include logarithmic, polynomial, rational, and radical functions. The course contains standards from the High School Conceptual Categories, each of which were written to encompass the scope of content and skills to be addressed throughout grades 9–12 not in any single course. Therefore, the full standard is presented in each model course, with clarifying footnotes as needed to limit the scope of the standard and indicate what is appropriate for study in a particular course. Standards that were limited in Algebra I, no longer have those restrictions in Algebra II. Students work closely with the expressions that define the functions, are facile with algebraic manipulations of expressions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Standards for Mathematical Practice apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas for this course are as follows:

- (1) A central theme is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students explore the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Connections are made between multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The Fundamental Theorem of Algebra is examined.
- (2) Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.
- (3) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*” is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.
- (4) Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

# Algebra II Standards

## Algebra II Overview

### Number and Quantity

#### The Real Number System

- Extend the properties of exponents to rational exponents.

#### Complex Number System

- Perform arithmetic operations with complex numbers
- Use complex numbers in polynomial identities and equations.

#### Vector and Matrix Quantities

- Represent and model with vector quantities
- Perform operations on matrices and use matrices in applications.

### Algebra

#### Seeing Structure in Expressions

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

#### Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems
- Rewrite rational expressions.

#### Creating Equations

- Create equations that describe numbers or equations.

#### Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning.
- Represent and solve equations and inequalities graphically.

### Functions

#### Interpreting Functions

- Interpret functions that arise in applications in terms of a context.
- Analyze functions using different representations.

#### Building Functions

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

#### Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems.

### Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### Functions (continued)

#### Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

### Statistics and Probability

#### Interpreting Categorical and Quantitative Data

- Summarize, represent and interpret data on a single count of measurement variable.

#### Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments and observational studies.

# Algebra II Standards

## Number and Quantity

### The Real Number System

N.RN

#### Extend the properties of exponents to rational exponents.<sup>1</sup>

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3) \times 3}$  to hold, so  $(5^{1/3})^3$  must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

### The Complex Number System

N.CN

#### Perform arithmetic operations with complex numbers.

1. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
2. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

#### Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .*
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

### Vector Quantities and Matrices

N.VM

#### Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $v$ ).
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

#### Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
12. (+) Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

## Algebra

### Seeing Structure in Expressions

A.SSE

#### Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. <sup>★</sup>
2. Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

#### Write expressions in equivalent forms to solve problems.

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* <sup>★</sup>

### Arithmetic with Polynomials and Rational Expressions

A.APR

#### Perform arithmetic operations on polynomials.

<sup>1</sup> Introduce rational exponents in simple situations; master in Algebra II

<sup>★</sup> Specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol appearing on the cluster heading should be understood to indicate that all standards in that cluster are modeling standards.

# Algebra II Standards

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

MA.1a. Divide polynomials.

## Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

## Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.*
5. (+) Know and apply that the Binomial Theorem gives the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.<sup>2</sup>

## Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

## Creating Equations★

A.CED

### Create equations that describe numbers or relationships. ★

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* ★
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .* ★

## Reasoning with equations and inequalities

A.REI

### Understand solving equations as a process of reasoning and explain the reasoning.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

### Represent and solve equations and inequalities graphically.

11. Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★

## Functions

### Interpreting Functions

F.IF

<sup>2</sup> The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

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# Algebra II Standards

## Interpret functions that arise in applications in terms of the context. ★

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.* ★
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

## Analyze functions using different representations.

- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
  - Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★
  - Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ★
  - Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ★
- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.  
MA.8c. Translate between different representations of functions and relations: graphs, equations, point sets, and tables.
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

## Building Functions

F.BF

### Build a function that models a relationship between two quantities.

- Write a function that describes a relationship between two quantities. ★
  - Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

### Build new functions from existing functions.

- Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- Find inverse functions.
  - Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. *For example,  $f(x) = 2(x^3)$  or  $f(x) = (x+1)/(x-1)$  for  $x \neq 1$ .*

## Linear, Quadratic, and Exponential Models★

F.LE

### Construct and compare linear, quadratic, and exponential models and solve problems. ★

- For exponential models, express as a logarithm the solution to  $ab^{(ct)} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology. ★

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# Algebra II Standards

## Trigonometric Functions

F.TF

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### Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

### Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★

### Prove and apply trigonometric identities.

8. Prove the Pythagorean identity  $(\sin A)^2 + (\cos A)^2 = 1$  and use it to find  $\sin A$ ,  $\cos A$ , or  $\tan A$ , given  $\sin A$ ,  $\cos A$ , or  $\tan A$ , and the quadrant of the angle.

## Statistics and Probability★

### Interpreting Categorical and Quantitative Data★

S.ID

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### Summarize, represent, and interpret data on a single count or measurement variable. ★

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★

### Making Inferences and Justifying Conclusions

S.IC

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### Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★

### Make inferences and justify conclusions from sample surveys, experiments and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★
6. Evaluate reports based on data. ★